

SETS

Exercise 1.1 page: 4

1. Which of the following are sets? Justify your answer.

- (i) The collection of all months of a year beginning with the letter J.
- (ii) The collection of ten most talented writers of India.
- (iii) A team of eleven best-cricket batsmen of the world.
- (iv) The collection of all boys in your class.
- (v) The collection of all natural numbers less than 100.
- (vi) A collection of novels written by the writer Munshi Prem Chand.
- (vii) The collection of all even integers.
- (viii) The collection of questions in this Chapter.
- (ix) A collection of most dangerous animals of the world.

Solution:

(i) The collection of all months of a year beginning with the letter J is a well-defined collection of objects as one can identify a month which belongs to this collection.

Therefore, this collection is a set.

(ii) The collection of ten most talented writers of India is not a well-defined collection as the criteria to determine a writer's talent may differ from one person to another.

Therefore, this collection is not a set.

(iii) A team of eleven best-cricket batsmen of the world is not a well-defined collection as the criteria to determine a batsman's talent may vary from one person to another.

Therefore, this collection is not a set.

(iv) The collection of all boys in your class is a well-defined collection as you can identify a boy who belongs to this collection.

Therefore, this collection is a set.

(v) The collection of all natural numbers less than 100 is a well-defined collection as one can find a number which belongs to this collection.

Therefore, this collection is a set.

(vi) A collection of novels written by the writer Munshi Prem Chand is a well-defined collection as one can find any book which belongs to this collection.

Therefore, this collection is a set.

(vii) The collection of all even integers is a well-defined collection as one can find an integer which belongs to this collection.

Therefore, this collection is a set.

(viii) The collection of questions in this chapter is a well-defined collection as one can find a question which belongs to this chapter.

Therefore, this collection is a set.

(ix) A collection of most dangerous animals of the world is not a well-defined collection as the criteria to find the dangerousness of an animal can differ from one animal to another.

Therefore, this collection is not a set.

2. Let $A = \{1, 2, 3, 4, 5, 6\}$. Insert the appropriate symbol \in or \notin in the blank spaces:

- (i) $5 \dots A$ (ii) $8 \dots A$ (iii) $0 \dots A$
- (iv) $4 \dots A$ (v) $2 \dots A$ (vi) $10 \dots A$

Solution:

- (i) $5 \in A$
- (ii) $8 \notin A$
- (iii) $0 \notin A$
- (iv) $4 \in A$
- (v) $2 \in A$
- (vi) $10 \notin A$

3. Write the following sets in roster form:

- (i) $A = \{x : x \text{ is an integer and } -3 < x < 7\}$.
- (ii) $B = \{x : x \text{ is a natural number less than } 6\}$.
- (iii) $C = \{x : x \text{ is a two-digit natural number such that the sum of its digits is } 8\}$
- (iv) $D = \{x : x \text{ is a prime number which is divisor of } 60\}$.
- (v) $E = \text{The set of all letters in the word TRIGONOMETRY.}$
- (vi) $F = \text{The set of all letters in the word BETTER.}$

Solution:

- (i) $A = \{x : x \text{ is an integer and } -3 < x < 7\}$
 $-2, -1, 0, 1, 2, 3, 4, 5, \text{ and } 6$ only are the elements of this set.

Hence, the given set can be written in roster form as

$$A = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$$

- (ii) $B = \{x : x \text{ is a natural number less than } 6\}$
 $1, 2, 3, 4, \text{ and } 5$ only are the elements of this set

Hence, the given set can be written in roster form as

$$B = \{1, 2, 3, 4, 5\}$$

(iii) $C = \{x : x \text{ is a two-digit natural number such that the sum of its digits is 8}\}$

17, 26, 35, 44, 53, 62, 71, and 80 only are the elements of this set

Hence, the given set can be written in roster form as

$$C = \{17, 26, 35, 44, 53, 62, 71, 80\}$$

(iv) $D = \{x : x \text{ is a prime number which is divisor of 60}\}$

2	60
2	30
3	15
	5

Here $60 = 2 \times 2 \times 3 \times 5$

2, 3 and 5 only are the elements of this set

Hence, the given set can be written in roster form as

$$D = \{2, 3, 5\}$$

(v) $E = \text{The set of all letters in the word TRIGONOMETRY}$

TRIGONOMETRY is a 12 letters word out of which T, R and O are repeated.

Hence, the given set can be written in roster form as

$$E = \{T, R, I, G, O, N, M, E, Y\}$$

(vi) $F = \text{The set of all letters in the word BETTER}$

BETTER is a 6 letters word out of which E and T are repeated.

Hence, the given set can be written in roster form as

$$F = \{B, E, T, R\}$$

4. Write the following sets in the set-builder form:

(i) $\{3, 6, 9, 12\}$

(ii) $\{2, 4, 8, 16, 32\}$

(iii) $\{5, 25, 125, 625\}$

(iv) $\{2, 4, 6 \dots\}$

(v) $\{1, 4, 9 \dots 100\}$

Solution:

(i) $\{3, 6, 9, 12\}$

The given set can be written in the set-builder form as $\{x : x = 3n, n \in \mathbb{N} \text{ and } 1 \leq n \leq 4\}$

(ii) $\{2, 4, 8, 16, 32\}$

We know that $2 = 2^1, 4 = 2^2, 8 = 2^3, 16 = 2^4$, and $32 = 2^5$.

Therefore, the given set $\{2, 4, 8, 16, 32\}$ can be written in the set-builder form as $\{x : x = 2^n, n \in \mathbb{N} \text{ and } 1 \leq n \leq 5\}$.

(iii) $\{5, 25, 125, 625\}$

We know that $5 = 5^1, 25 = 5^2, 125 = 5^3$, and $625 = 5^4$.

Therefore, the given set $\{5, 25, 125, 625\}$ can be written in the set-builder form as $\{x: x = 5^n, n \in \mathbb{N} \text{ and } 1 \leq n \leq 4\}$.

(iv) $\{2, 4, 6 \dots\}$

$\{2, 4, 6 \dots\}$ is a set of all even natural numbers

Therefore, the given set $\{2, 4, 6 \dots\}$ can be written in the set-builder form as $\{x: x \text{ is an even natural number}\}$.

(v) $\{1, 4, 9 \dots 100\}$

We know that $1 = 1^2, 4 = 2^2, 9 = 3^2 \dots 100 = 10^2$.

Therefore, the given set $\{1, 4, 9 \dots 100\}$ can be written in the set-builder form as $\{x: x = n^2, n \in \mathbb{N} \text{ and } 1 \leq n \leq 10\}$.

5. List all the elements of the following sets:

(i) $A = \{x: x \text{ is an odd natural number}\}$

(ii) $B = \{x: x \text{ is an integer, } -1/2 < x < 9/2\}$

(iii) $C = \{x: x \text{ is an integer, } x^2 \leq 4\}$

(iv) $D = \{x: x \text{ is a letter in the word "LOYAL"}\}$

(v) $E = \{x: x \text{ is a month of a year not having 31 days}\}$

(vi) $F = \{x: x \text{ is a consonant in the English alphabet which proceeds } k\}$.

Solution:

(i) $A = \{x: x \text{ is an odd natural number}\}$

So the elements are $A = \{1, 3, 5, 7, 9 \dots\}$

(ii) $B = \{x: x \text{ is an integer, } -1/2 < x < 9/2\}$

We know that $-1/2 = -0.5$ and $9/2 = 4.5$

So the elements are $B = \{0, 1, 2, 3, 4\}$.

(iii) $C = \{x: x \text{ is an integer, } x^2 \leq 4\}$

We know that

$(-1)^2 = 1 \leq 4; (-2)^2 = 4 \leq 4; (-3)^2 = 9 > 4$

Here

$0^2 = 0 \leq 4, 1^2 = 1 \leq 4, 2^2 = 4 \leq 4, 3^2 = 9 > 4$

So we get

$C = \{-2, -1, 0, 1, 2\}$

(iv) $D = \{x: x \text{ is a letter in the word "LOYAL"}\}$

So the elements are $D = \{L, O, Y, A\}$

(v) $E = \{x: x \text{ is a month of a year not having 31 days}\}$

So the elements are $E = \{\text{February, April, June, September, November}\}$

(vi) $F = \{x: x \text{ is a consonant in the English alphabet which proceeds } k\}$

So the elements are $F = \{b, c, d, f, g, h, j\}$

6. Match each of the set on the left in the roster form with the same set on the right described in set-builder form:

- (i) $\{1, 2, 3, 6\}$ (a) $\{x: x \text{ is a prime number and a divisor of } 6\}$
- (ii) $\{2, 3\}$ (b) $\{x: x \text{ is an odd natural number less than } 10\}$
- (iii) $\{M, A, T, H, E, I, C, S\}$ (c) $\{x: x \text{ is a natural number and divisor of } 6\}$
- (iv) $\{1, 3, 5, 7, 9\}$ (d) $\{x: x \text{ is a letter of the word MATHEMATICS}\}$

Solution:

- (i) Here the elements of this set are natural number as well as divisors of 6. Hence, (i) matches with (c).
- (ii) 2 and 3 are prime numbers which are divisors of 6. Hence, (ii) matches with (a).
- (iii) The elements are the letters of the word MATHEMATICS. Hence, (iii) matches with (d).
- (iv) The elements are odd natural numbers which are less than 10. Hence, (v) matches with (b).

Exercise 1.2 page: 8

1. Which of the following are examples of the null set?

- (i) Set of odd natural numbers divisible by 2
- (ii) Set of even prime numbers
- (iii) $\{x: x \text{ is a natural numbers, } x < 5 \text{ and } x > 7\}$
- (iv) $\{y: y \text{ is a point common to any two parallel lines}\}$

Solution:

- (i) Set of odd natural numbers divisible by 2 is a null set as odd numbers are not divisible by 2.
- (ii) Set of even prime numbers is not a null set as 2 is an even prime number.
- (iii) $\{x: x \text{ is a natural number, } x < 5 \text{ and } x > 7\}$ is a null set as a number cannot be both less than 5 and greater than 7.
- (iv) $\{y: y \text{ is a point common to any two parallel lines}\}$ is a null set as the parallel lines do not intersect. Therefore, they have no common point.

2. Which of the following sets are finite or infinite?

- (i) The set of months of a year
- (ii) $\{1, 2, 3 \dots\}$
- (iii) $\{1, 2, 3 \dots 99, 100\}$
- (iv) The set of positive integers greater than 100
- (v) The set of prime numbers less than 99

Solution:

- (i) The set of months of a year is a finite set as it contains 12 elements.
- (ii) $\{1, 2, 3 \dots\}$ is an infinite set because it has infinite number of natural numbers.
- (iii) $\{1, 2, 3 \dots 99, 100\}$ is a finite set as the numbers from 1 to 100 are finite.
- (iv) The set of positive integers greater than 100 is an infinite set as the positive integers which are greater than 100 are infinite.
- (v) The set of prime numbers less than 99 is a finite set as the prime numbers which are less than 99 are finite.

3. State whether each of the following set is finite or infinite:

- (i) The set of lines which are parallel to the x -axis
- (ii) The set of letters in the English alphabet
- (iii) The set of numbers which are multiple of 5
- (iv) The set of animals living on the earth
- (v) The set of circles passing through the origin $(0, 0)$

Solution:

- (i) The set of lines which are parallel to the x -axis is an infinite set as the lines which are parallel to the x -axis are infinite.
- (ii) The set of letters in the English alphabet is a finite set as it contains 26 elements.
- (iii) The set of numbers which are multiple of 5 is an infinite set as the multiples of 5 are infinite.
- (iv) The set of animals living on the earth is a finite set as the number of animals living on the earth is finite.
- (v) The set of circles passing through the origin $(0, 0)$ is an infinite set as infinite number of circles can pass through the origin.

4. In the following, state whether $A = B$ or not:

- (i) $A = \{a, b, c, d\}; B = \{d, c, b, a\}$
- (ii) $A = \{4, 8, 12, 16\}; B = \{8, 4, 16, 18\}$
- (iii) $A = \{2, 4, 6, 8, 10\}; B = \{x: x \text{ is positive even integer and } x \leq 10\}$
- (iv) $A = \{x: x \text{ is a multiple of 10}\}; B = \{10, 15, 20, 25, 30 \dots\}$

Solution:

- (i) $A = \{a, b, c, d\}; B = \{d, c, b, a\}$

Order in which the elements of a set are listed is not significant.

Therefore, $A = B$.

- (ii) $A = \{4, 8, 12, 16\}; B = \{8, 4, 16, 18\}$

We know that $12 \in A$ but $12 \notin B$.

Therefore, $A \neq B$

(iii) $A = \{2, 4, 6, 8, 10\}$;

$B = \{x : x \text{ is a positive even integer and } x \leq 10\} = \{2, 4, 6, 8, 10\}$

Therefore, $A = B$

(iv) $A = \{x : x \text{ is a multiple of } 10\}$

$B = \{10, 15, 20, 25, 30, \dots\}$

We know that $15 \in B$ but $15 \notin A$.

Therefore, $A \neq B$

5. Are the following pair of sets equal? Give reasons.

(i) $A = \{2, 3\}$; $B = \{x : x \text{ is solution of } x^2 + 5x + 6 = 0\}$

(ii) $A = \{x : x \text{ is a letter in the word FOLLOW}\}$; $B = \{y : y \text{ is a letter in the word WOLF}\}$

Solution:

(i) $A = \{2, 3\}$; $B = \{x : x \text{ is solution of } x^2 + 5x + 6 = 0\}$

$x^2 + 5x + 6 = 0$ can be written as

$$x(x+3) + 2(x+3) = 0$$

By further calculation

$$(x+2)(x+3) = 0$$

So we get

$$x = -2 \text{ or } x = -3$$

Here

$A = \{2, 3\}$; $B = \{-2, -3\}$

Therefore, $A \neq B$

(ii) $A = \{x : x \text{ is a letter in the word FOLLOW}\} = \{F, O, L, W\}$

$B = \{y : y \text{ is a letter in the word WOLF}\} = \{W, O, L, F\}$

Order in which the elements of a set which are listed is not significant.

Therefore, $A = B$.

6. From the sets given below, select equal sets:

$A = \{2, 4, 8, 12\}$, $B = \{1, 2, 3, 4\}$, $C = \{4, 8, 12, 14\}$, $D = \{3, 1, 4, 2\}$

$E = \{-1, 1\}$, $F = \{0, a\}$, $G = \{1, -1\}$, $H = \{0, 1\}$

Solution:

$A = \{2, 4, 8, 12\}$; $B = \{1, 2, 3, 4\}$; $C = \{4, 8, 12, 14\}$

$D = \{3, 1, 4, 2\}$; $E = \{-1, 1\}$; $F = \{0, a\}$

$G = \{1, -1\}$; $H = \{0, 1\}$

We know that

$8 \in A, 8 \notin B, 8 \notin D, 8 \notin E, 8 \notin F, 8 \notin G, 8 \notin H$

$A \neq B, A \neq D, A \neq E, A \neq F, A \neq G, A \neq H$

It can be written as

$2 \in A, 2 \notin C$

Therefore, $A \neq C$

$3 \in B, 3 \notin C, 3 \notin E, 3 \notin F, 3 \notin G, 3 \notin H$

$B \neq C, B \neq E, B \neq F, B \neq G, B \neq H$

It can be written as

$12 \in C, 12 \notin D, 12 \notin E, 12 \notin F, 12 \notin G, 12 \notin H$

Therefore, $C \neq D, C \neq E, C \neq F, C \neq G, C \neq H$

$4 \in D, 4 \notin E, 4 \notin F, 4 \notin G, 4 \notin H$

Therefore, $D \neq E, D \neq F, D \neq G, D \neq H$

Here, $E \neq F, E \neq G, E \neq H$

$F \neq G, F \neq H, G \neq H$

Order in which the elements of a set are listed is not significant.

$B = D$ and $E = G$

Therefore, among the given sets, $B = D$ and $E = G$.

Exercise 1.3 page: 12

1. Make correct statements by filling in the symbols \subset or $\not\subset$ in the blank spaces:

(i) $\{2, 3, 4\} \dots \{1, 2, 3, 4, 5\}$

(ii) $\{a, b, c\} \dots \{b, c, d\}$

(iii) $\{x: x \text{ is a student of Class XI of your school}\} \dots \{x: x \text{ student of your school}\}$

(iv) $\{x: x \text{ is a circle in the plane}\} \dots \{x: x \text{ is a circle in the same plane with radius 1 unit}\}$

(v) $\{x: x \text{ is a triangle in a plane}\} \dots \{x: x \text{ is a rectangle in the plane}\}$

(vi) $\{x: x \text{ is an equilateral triangle in a plane}\} \dots \{x: x \text{ is a triangle in the same plane}\}$

(vii) $\{x: x \text{ is an even natural number}\} \dots \{x: x \text{ is an integer}\}$

Solution:

(i) $\{2, 3, 4\} \subset \{1, 2, 3, 4, 5\}$

(ii) $\{a, b, c\} \not\subset \{b, c, d\}$

(iii) $\{x: x \text{ is a student of Class XI of your school}\} \subset \{x: x \text{ student of your school}\}$

(iv) $\{x: x \text{ is a circle in the plane}\} \not\subset \{x: x \text{ is a circle in the same plane with radius 1 unit}\}$

(v) $\{x: x \text{ is a triangle in a plane}\} \not\subset \{x: x \text{ is a rectangle in the plane}\}$

(vi) $\{x: x \text{ is an equilateral triangle in a plane}\} \subset \{x: x \text{ is a triangle in the same plane}\}$

(vii) $\{x: x \text{ is an even natural number}\} \subset \{x: x \text{ is an integer}\}$

2. Examine whether the following statements are true or false:

- (i) $\{a, b\} \not\subset \{b, c, a\}$
- (ii) $\{a, e\} \subset \{x: x \text{ is a vowel in the English alphabet}\}$
- (iii) $\{1, 2, 3\} \subset \{1, 3, 5\}$
- (iv) $\{a\} \subset \{a, b, c\}$
- (v) $\{a\} \in (a, b, c)$

(vi) $\{x: x \text{ is an even natural number less than } 6\} \subset \{x: x \text{ is a natural number which divides } 36\}$

Solution:

(i) False.

Here each element of $\{a, b\}$ is an element of $\{b, c, a\}$.

(ii) True.

We know that a, e are two vowels of the English alphabet.

(iii) False.

$2 \in \{1, 2, 3\}$ where, $2 \notin \{1, 3, 5\}$

(iv) True.

Each element of $\{a\}$ is also an element of $\{a, b, c\}$.

(v) False.

Elements of $\{a, b, c\}$ are a, b, c . Hence, $\{a\} \subset \{a, b, c\}$

(vi) True.

$\{x: x \text{ is an even natural number less than } 6\} = \{2, 4\}$

$\{x: x \text{ is a natural number which divides } 36\} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

3. Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are incorrect and why?

- (i) $\{3, 4\} \subset A$
- (ii) $\{3, 4\} \in A$
- (iii) $\{\{3, 4\}\} \subset A$
- (iv) $1 \in A$
- (v) $1 \subset A$
- (vi) $\{1, 2, 5\} \subset A$
- (vii) $\{1, 2, 5\} \in A$
- (viii) $\{1, 2, 3\} \subset A$
- (ix) $\Phi \in A$

(x) $\Phi \subset A$

(xi) $\{\Phi\} \subset A$

Solution:

It is given that $A = \{1, 2, \{3, 4\}, 5\}$

(i) $\{3, 4\} \subset A$ is incorrect

Here $3 \in \{3, 4\}$; where, $3 \notin A$.

(ii) $\{3, 4\} \in A$ is correct

$\{3, 4\}$ is an element of A .

(iii) $\{\{3, 4\}\} \subset A$ is correct

$\{3, 4\} \in \{\{3, 4\}\}$ and $\{3, 4\} \in A$.

(iv) $1 \in A$ is correct

1 is an element of A .

(v) $1 \subset A$ is incorrect

An element of a set can never be a subset of itself.

(vi) $\{1, 2, 5\} \subset A$ is correct

Each element of $\{1, 2, 5\}$ is also an element of A .

(vii) $\{1, 2, 5\} \in A$ is incorrect

$\{1, 2, 5\}$ is not an element of A .

(viii) $\{1, 2, 3\} \subset A$ is incorrect

$3 \in \{1, 2, 3\}$; where, $3 \notin A$.

(ix) $\Phi \in A$ is incorrect

Φ is not an element of A .

(x) $\Phi \subset A$ is correct

Φ is a subset of every set.

(xi) $\{\Phi\} \subset A$ is incorrect

$\Phi \in \{\Phi\}$; where, $\Phi \in A$.

4. Write down all the subsets of the following sets:

(i) $\{a\}$

(ii) $\{a, b\}$

(iii) $\{1, 2, 3\}$

(iv) Φ

Solution:

(i) Subsets of $\{a\}$ are

Φ and $\{a\}$.

(ii) Subsets of $\{a, b\}$ are

$\Phi, \{a\}, \{b\}$, and $\{a, b\}$.

(iii) Subsets of $\{1, 2, 3\}$ are

Call :- + 91 9953771000

@ 1/354, SADAR BAZAR, DELHI, CANTT - 110010

$\Phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}$, and $\{1, 2, 3\}$.

(iv) Only subset of Φ is Φ .

5. How many elements has $P(A)$, if $A = \Phi$?

Solution:

If A is a set with m elements

$n(A) = m$ then $n[P(A)] = 2^m$

If $A = \Phi$ we get $n(A) = 0$

$n[P(A)] = 2^0 = 1$

Therefore, $P(A)$ has one element.

6. Write the following as intervals:

(i) $\{x: x \in \mathbb{R}, -4 < x \leq 6\}$

(ii) $\{x: x \in \mathbb{R}, -12 < x < -10\}$

(iii) $\{x: x \in \mathbb{R}, 0 \leq x < 7\}$

(iv) $\{x: x \in \mathbb{R}, 3 \leq x \leq 4\}$

Solution:

(i) $\{x: x \in \mathbb{R}, -4 < x \leq 6\} = (-4, 6]$

(ii) $\{x: x \in \mathbb{R}, -12 < x < -10\} = (-12, -10)$

(iii) $\{x: x \in \mathbb{R}, 0 \leq x < 7\} = [0, 7)$

(iv) $\{x: x \in \mathbb{R}, 3 \leq x \leq 4\} = [3, 4]$

7. Write the following intervals in set-builder form:

(i) $(-3, 0)$

(ii) $[6, 12]$

(iii) $(6, 12]$

(iv) $[-23, 5)$

Solution:

(i) $(-3, 0) = \{x: x \in \mathbb{R}, -3 < x < 0\}$

(ii) $[6, 12] = \{x: x \in \mathbb{R}, 6 \leq x \leq 12\}$

(iii) $(6, 12] = \{x: x \in \mathbb{R}, 6 < x \leq 12\}$

(iv) $[-23, 5) = \{x: x \in \mathbb{R}, -23 \leq x < 5\}$

8. What universal set (s) would you propose for each of the following?

(i) The set of right triangles

(ii) The set of isosceles triangles

Solution:

(i) Among the set of right triangles, the universal set is the set of triangles or the set of polygons.

(ii) Among the set of isosceles triangles, the universal set is the set of triangles or the set of polygons or the set of two-dimensional figures.

9. Given the sets $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$, which of the following may be considered as universal set (s) for all the three sets A, B and C?

- (i) $\{0, 1, 2, 3, 4, 5, 6\}$
- (ii) Φ
- (iii) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- (iv) $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Solution:

(i) We know that $A \subset \{0, 1, 2, 3, 4, 5, 6\}$

$B \subset \{0, 1, 2, 3, 4, 5, 6\}$

So $C \not\subset \{0, 1, 2, 3, 4, 5, 6\}$

Hence, the set $\{0, 1, 2, 3, 4, 5, 6\}$ cannot be the universal set for the sets A, B, and C.

(ii) $A \not\subset \Phi$, $B \not\subset \Phi$, $C \not\subset \Phi$

Hence, Φ cannot be the universal set for the sets A, B, and C.

(iii) $A \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$B \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$C \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Hence, the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is the universal set for the sets A, B, and C.

(iv) $A \subset \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B \subset \{1, 2, 3, 4, 5, 6, 7, 8\}$

So $C \not\subset \{1, 2, 3, 4, 5, 6, 7, 8\}$

Hence, the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ cannot be the universal set for the sets A, B, and C.

Exercise 1.4 page: 17

1. Find the union of each of the following pairs of sets:

(i) $X = \{1, 3, 5\}$ $Y = \{1, 2, 3\}$

(ii) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$

(iii) $A = \{x: x \text{ is a natural number and multiple of 3}\}$

$B = \{x: x \text{ is a natural number less than 6}\}$

(iv) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\}$

$B = \{x: x \text{ is a natural number and } 6 < x < 10\}$

(v) $A = \{1, 2, 3\}$, $B = \Phi$

Solution:

(i) $X = \{1, 3, 5\}$ $Y = \{1, 2, 3\}$

So the union of the pairs of set can be written as

$$X \cup Y = \{1, 2, 3, 5\}$$

(ii) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$

So the union of the pairs of set can be written as

$$A \cup B = \{a, b, c, e, i, o, u\}$$

(iii) $A = \{x : x \text{ is a natural number and multiple of } 3\} = \{3, 6, 9, \dots\}$

$B = \{x : x \text{ is a natural number less than } 6\} = \{1, 2, 3, 4, 5, 6\}$

So the union of the pairs of set can be written as

$$A \cup B = \{1, 2, 4, 5, 3, 6, 9, 12, \dots\}$$

Hence, $A \cup B = \{x : x = 1, 2, 4, 5 \text{ or a multiple of } 3\}$

(iv) $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\} = \{2, 3, 4, 5, 6\}$

$B = \{x : x \text{ is a natural number and } 6 < x < 10\} = \{7, 8, 9\}$

So the union of the pairs of set can be written as

$$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$$

Hence, $A \cup B = \{x : x \in \mathbb{N} \text{ and } 1 < x < 10\}$

(v) $A = \{1, 2, 3\}$, $B = \emptyset$

So the union of the pairs of set can be written as

$$A \cup B = \{1, 2, 3\}$$

2. Let $A = \{a, b\}$, $B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?

Solution:

It is given that

$$A = \{a, b\} \text{ and } B = \{a, b, c\}$$

Yes, $A \subset B$

So the union of the pairs of set can be written as

$$A \cup B = \{a, b, c\} = B$$

3. If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Solution:

If A and B are two sets such that $A \subset B$, then $A \cup B = B$.

4. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$; find

(i) $A \cup B$

(ii) $A \cup C$

(iii) $B \cup C$

(iv) $B \cup D$

(v) $A \cup B \cup C$

(vi) $A \cup B \cup D$

(vii) $B \cup C \cup D$

Solution:

It is given that

$A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$

- (i) $A \cup B = \{1, 2, 3, 4, 5, 6\}$
- (ii) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- (iii) $B \cup C = \{3, 4, 5, 6, 7, 8\}$
- (iv) $B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$
- (v) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- (vi) $A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- (vii) $B \cup C \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

5. Find the intersection of each pair of sets:

- (i) $X = \{1, 3, 5\}$, $Y = \{1, 2, 3\}$
- (ii) $A = \{a, e, i, o, u\}$, $B = \{a, b, c\}$
- (iii) $A = \{x: x \text{ is a natural number and multiple of 3}\}$
 $B = \{x: x \text{ is a natural number less than 6}\}$
- (iv) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\}$
 $B = \{x: x \text{ is a natural number and } 6 < x < 10\}$
- (v) $A = \{1, 2, 3\}$, $B = \Phi$

Solution:

- (i) $X = \{1, 3, 5\}$, $Y = \{1, 2, 3\}$

So the intersection of the given set can be written as

$$X \cap Y = \{1, 3\}$$

- (ii) $A = \{a, e, i, o, u\}$, $B = \{a, b, c\}$

So the intersection of the given set can be written as

$$A \cap B = \{a\}$$

- (iii) $A = \{x: x \text{ is a natural number and multiple of 3}\} = \{3, 6, 9, \dots\}$

$$B = \{x: x \text{ is a natural number less than 6}\} = \{1, 2, 3, 4, 5\}$$

So the intersection of the given set can be written as

$$A \cap B = \{3\}$$

- (iv) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\} = \{2, 3, 4, 5, 6\}$

$$B = \{x: x \text{ is a natural number and } 6 < x < 10\} = \{7, 8, 9\}$$

So the intersection of the given set can be written as

$$A \cap B = \Phi$$

- (v) $A = \{1, 2, 3\}$, $B = \Phi$

So the intersection of the given set can be written as

$$A \cap B = \Phi$$

6. If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$; find

- (i) $A \cap B$
- (ii) $B \cap C$
- (iii) $A \cap C \cap D$
- (iv) $A \cap C$
- (v) $B \cap D$
- (vi) $A \cap (B \cup C)$
- (vii) $A \cap D$
- (viii) $A \cap (B \cup D)$
- (ix) $(A \cap B) \cap (B \cup C)$
- (x) $(A \cup D) \cap (B \cup C)$

Solution:

$$\begin{aligned} \text{(i) } A \cap B &= \{7, 9, 11\} \\ \text{(ii) } B \cap C &= \{11, 13\} \\ \text{(iii) } A \cap C \cap D &= \{A \cap C\} \cap D \\ &= \{11\} \cap \{15, 17\} \\ &= \Phi \end{aligned}$$

$$\text{(iv) } A \cap C = \{11\}$$

$$\text{(v) } B \cap D = \Phi$$

$$\begin{aligned} \text{(vi) } A \cap (B \cup C) &= (A \cap B) \cup (A \cap C) \\ &= \{7, 9, 11\} \cup \{11\} \\ &= \{7, 9, 11\} \end{aligned}$$

$$\text{(vii) } A \cap D = \Phi$$

$$\begin{aligned} \text{(viii) } A \cap (B \cup D) &= (A \cap B) \cup (A \cap D) \\ &= \{7, 9, 11\} \cup \Phi \\ &= \{7, 9, 11\} \end{aligned}$$

$$\begin{aligned} \text{(ix) } (A \cap B) \cap (B \cup C) &= \{7, 9, 11\} \cap \{7, 9, 11, 13, 15\} \\ &= \{7, 9, 11\} \end{aligned}$$

$$\begin{aligned} \text{(x) } (A \cup D) \cap (B \cup C) &= \{3, 5, 7, 9, 11, 15, 17\} \cap \{7, 9, 11, 13, 15\} \\ &= \{7, 9, 11, 15\} \end{aligned}$$

7. If $A = \{x: x \text{ is a natural number}\}$, $B = \{x: x \text{ is an even natural number}\}$, $C = \{x: x \text{ is an odd natural number}\}$ and $D = \{x: x \text{ is a prime number}\}$, find

- (i) $A \cap B$
- (ii) $A \cap C$
- (iii) $A \cap D$
- (iv) $B \cap C$

(v) $B \cap D$ (vi) $C \cap D$ **Solution:**

It can be written as

$$A = \{x: x \text{ is a natural number}\} = \{1, 2, 3, 4, 5, \dots\}$$

$$B = \{x: x \text{ is an even natural number}\} = \{2, 4, 6, 8, \dots\}$$

$$C = \{x: x \text{ is an odd natural number}\} = \{1, 3, 5, 7, 9, \dots\}$$

$$D = \{x: x \text{ is a prime number}\} = \{2, 3, 5, 7, \dots\}$$

$$(i) A \cap B = \{x: x \text{ is an even natural number}\} = B$$

$$(ii) A \cap C = \{x: x \text{ is an odd natural number}\} = C$$

$$(iii) A \cap D = \{x: x \text{ is a prime number}\} = D$$

$$(iv) B \cap C = \emptyset$$

$$(v) B \cap D = \{2\}$$

$$(vi) C \cap D = \{x: x \text{ is an odd prime number}\}$$

8. Which of the following pairs of sets are disjoint?

$$(i) \{1, 2, 3, 4\} \text{ and } \{x: x \text{ is a natural number and } 4 \leq x \leq 6\}$$

$$(ii) \{a, e, i, o, u\} \text{ and } \{c, d, e, f\}$$

$$(iii) \{x: x \text{ is an even integer}\} \text{ and } \{x: x \text{ is an odd integer}\}$$

Solution:

$$(i) \{1, 2, 3, 4\}$$

$$\{x: x \text{ is a natural number and } 4 \leq x \leq 6\} = \{4, 5, 6\}$$

So we get

$$\{1, 2, 3, 4\} \cap \{4, 5, 6\} = \{4\}$$

Hence, this pair of sets is not disjoint.

$$(ii) \{a, e, i, o, u\} \cap \{c, d, e, f\} = \{e\}$$

Hence, $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$ are not disjoint.

$$(iii) \{x: x \text{ is an even integer}\} \cap \{x: x \text{ is an odd integer}\} = \emptyset$$

Hence, this pair of sets is disjoint.

9. If $A = \{3, 6, 9, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$, **$C = \{2, 4, 6, 8, 10, 12, 14, 16\}$, $D = \{5, 10, 15, 20\}$; find**(i) $A - B$ (ii) $A - C$ (iii) $A - D$ (iv) $B - A$ (v) $C - A$ (vi) $D - A$ (vii) $B - C$

(viii) B - D

(ix) C - B

(x) D - B

(xi) C - D

(xii) D - C

Solution:

(i) $A - B = \{3, 6, 9, 15, 18, 21\}$

(ii) $A - C = \{3, 9, 15, 18, 21\}$

(iii) $A - D = \{3, 6, 9, 12, 18, 21\}$

(iv) $B - A = \{4, 8, 16, 20\}$

(v) $C - A = \{2, 4, 8, 10, 14, 16\}$

(vi) $D - A = \{5, 10, 20\}$

(vii) $B - C = \{20\}$

(viii) $B - D = \{4, 8, 12, 16\}$

(ix) $C - B = \{2, 6, 10, 14\}$

(x) $D - B = \{5, 10, 15\}$

(xi) $C - D = \{2, 4, 6, 8, 12, 14, 16\}$

(xii) $D - C = \{5, 15, 20\}$

10. If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find

(i) $X - Y$

(ii) $Y - X$

(iii) $X \cap Y$

Solution:

(i) $X - Y = \{a, c\}$

(ii) $Y - X = \{f, g\}$

(iii) $X \cap Y = \{b, d\}$

11. If R is the set of real numbers and Q is the set of rational numbers, then what is $R - Q$?

Solution:

We know that

R - Set of real numbers

Q - Set of rational numbers

Hence, $R - Q$ is a set of irrational numbers.

12. State whether each of the following statement is true or false. Justify your answer.

(i) $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint sets.

(ii) $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets.

(iii) $\{2, 6, 10, 14\}$ and $\{3, 7, 11, 15\}$ are disjoint sets.

(iv) $\{2, 6, 10\}$ and $\{3, 7, 11\}$ are disjoint sets.

Solution:

(i) False

If $3 \in \{2, 3, 4, 5\}$, $3 \in \{3, 6\}$

So we get $\{2, 3, 4, 5\} \cap \{3, 6\} = \{3\}$

(ii) False

If $a \in \{a, e, i, o, u\}$, $a \in \{a, b, c, d\}$

So we get $\{a, e, i, o, u\} \cap \{a, b, c, d\} = \{a\}$

(iii) True

Here $\{2, 6, 10, 14\} \cap \{3, 7, 11, 15\} = \emptyset$

(iv) True

Here $\{2, 6, 10\} \cap \{3, 7, 11\} = \emptyset$

Exercise 1.5 Page: 20

1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. Find

(i) A'

(ii) B'

(iii) $(A \cup C)'$

(iv) $(A \cup B)'$

(v) $(A')'$

(vi) $(B - C)'$

Solution:

It is given that

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 3, 4\}$

$B = \{2, 4, 6, 8\}$

$C = \{3, 4, 5, 6\}$

(i) $A' = \{5, 6, 7, 8, 9\}$

(ii) $B' = \{1, 3, 5, 7, 9\}$

(iii) $A \cup C = \{1, 2, 3, 4, 5, 6\}$

So we get

$(A \cup C)' = \{7, 8, 9\}$

(iv) $A \cup B = \{1, 2, 3, 4, 6, 8\}$

So we get

$$(A \cup B)' = \{5, 7, 9\}$$

$$(v) (A')' = A = \{1, 2, 3, 4\}$$

$$(vi) B - C = \{2, 8\}$$

So we get

$$(B - C)' = \{1, 3, 4, 5, 6, 7, 9\}$$

2. If $U = \{a, b, c, d, e, f, g, h\}$, find the complements of the following sets:

$$(i) A = \{a, b, c\}$$

$$(ii) B = \{d, e, f, g\}$$

$$(iii) C = \{a, c, e, g\}$$

$$(iv) D = \{f, g, h, a\}$$

=

Solution:

$$(i) A = \{a, b, c\}$$

So we get

$$A' = \{d, e, f, g, h\}$$

$$(ii) B = \{d, e, f, g\}$$

So we get

$$B' = \{a, b, c, h\}$$

$$(iii) C = \{a, c, e, g\}$$

So we get

$$C' = \{b, d, f, h\}$$

$$(iv) D = \{f, g, h, a\}$$

So we get

$$D' = \{b, c, d, e\}$$

3. Taking the set of natural numbers as the universal set, write down the complements of the following sets:

$$(i) \{x: x \text{ is an even natural number}\}$$

$$(ii) \{x: x \text{ is an odd natural number}\}$$

$$(iii) \{x: x \text{ is a positive multiple of 3}\}$$

$$(iv) \{x: x \text{ is a prime number}\}$$

$$(v) \{x: x \text{ is a natural number divisible by 3 and 5}\}$$

$$(vi) \{x: x \text{ is a perfect square}\}$$

$$(vii) \{x: x \text{ is a perfect cube}\}$$

$$(viii) \{x: x + 5 = 8\}$$

$$(ix) \{x: 2x + 5 = 9\}$$

$$(x) \{x: x \geq 7\}$$

$$(xi) \{x: x \in \mathbb{N} \text{ and } 2x + 1 > 10\}$$

Solution:

We know that

$U = N$: Set of natural numbers

- (i) $\{x: x \text{ is an even natural number}\}' = \{x: x \text{ is an odd natural number}\}$
- (ii) $\{x: x \text{ is an odd natural number}\}' = \{x: x \text{ is an even natural number}\}$
- (iii) $\{x: x \text{ is a positive multiple of } 3\}' = \{x: x \in N \text{ and } x \text{ is not a multiple of } 3\}$
- (iv) $\{x: x \text{ is a prime number}\}' = \{x: x \text{ is a positive composite number and } x = 1\}$
- (v) $\{x: x \text{ is a natural number divisible by } 3 \text{ and } 5\}' = \{x: x \text{ is a natural number that is not divisible by } 3 \text{ or } 5\}$
- (vi) $\{x: x \text{ is a perfect square}\}' = \{x: x \in N \text{ and } x \text{ is not a perfect square}\}$
- (vii) $\{x: x \text{ is a perfect cube}\}' = \{x: x \in N \text{ and } x \text{ is not a perfect cube}\}$
- (viii) $\{x: x + 5 = 8\}' = \{x: x \in N \text{ and } x \neq 3\}$
- (ix) $\{x: 2x + 5 = 9\}' = \{x: x \in N \text{ and } x \neq 2\}$
- (x) $\{x: x \geq 7\}' = \{x: x \in N \text{ and } x < 7\}$
- (xi) $\{x: x \in N \text{ and } 2x + 1 > 10\}' = \{x: x \in N \text{ and } x \leq 9/2\}$

4. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$. Verify that

- (i) $(A \cup B)' = A' \cap B'$
- (ii) $(A \cap B)' = A' \cup B'$

Solution:

It is given that

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$A = \{2, 4, 6, 8\}$$

$$B = \{2, 3, 5, 7\}$$

$$(i) (A \cup B)' = \{2, 3, 4, 5, 6, 7, 8\}' = \{1, 9\}$$

$$A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\} = \{1, 9\}$$

Therefore, $(A \cup B)' = A' \cap B'$.

$$(ii) (A \cap B)' = \{2\}' = \{1, 3, 4, 5, 6, 7, 8, 9\}$$

$$A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\} = \{1, 3, 4, 5, 6, 7, 8, 9\}$$

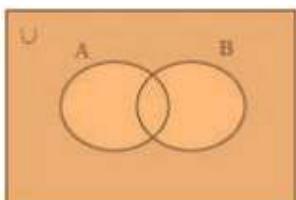
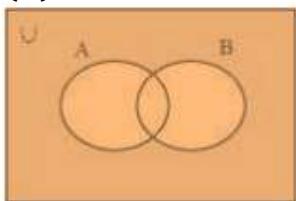
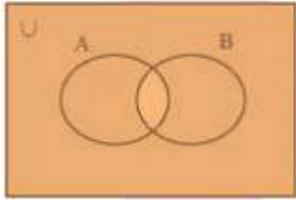
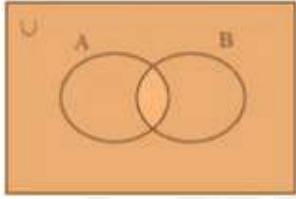
Therefore, $(A \cap B)' = A' \cup B'$.

5. Draw appropriate Venn diagram for each of the following:

- (i) $(A \cup B)'$
- (ii) $A' \cap B'$
- (iii) $(A \cap B)'$
- (iv) $A' \cup B'$

Solution:

$$(i) (A \cup B)'$$

(ii) $A' \cap B'$ (iii) $(A \cap B)'$ (iv) $A' \cup B'$ 

6. Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60° , what is A' ?

Solution:

A' is the set of all equilateral triangles.

7. Fill in the blanks to make each of the following a true statement:

(i) $A \cup A' = \dots$

(ii) $\Phi' \cap A = \dots$

(iii) $A \cap A' = \dots$

(iv) $U' \cap A = \dots$

Solution:

(i) $A \cup A' = U$

(ii) $\Phi' \cap A = U \cap A = A$

So we get

$\Phi' \cap A = A$

(iii) $A \cap A' = \Phi$

(iv) $U' \cap A = \Phi \cap A = \Phi$

So we get

$$U' \cap A = \emptyset$$

Exercise 1.6 page: 24

1. If X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$ and $n(X \cup Y) = 38$, find $n(X \cap Y)$.

Solution:

Given

$$n(X) = 17$$

$$n(Y) = 23$$

$$n(X \cup Y) = 38$$

We can write it as

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

Substituting the values

$$38 = 17 + 23 - n(X \cap Y)$$

By further calculation

$$n(X \cap Y) = 40 - 38 = 2$$

So we get

$$n(X \cap Y) = 2$$

2. If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements; how many elements does $X \cap Y$ have?

Solution:

Given

$$n(X \cup Y) = 18$$

$$n(X) = 8$$

$$n(Y) = 15$$

We can write it as

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

Substituting the values

$$18 = 8 + 15 - n(X \cap Y)$$

By further calculation

$$n(X \cap Y) = 23 - 18 = 5$$

So we get

$$n(X \cap Y) = 5$$

3. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

Solution:

Consider H as the set of people who speak Hindi
E as the set of people who speak English

We know that

$$n(H \cup E) = 400$$

$$n(H) = 250$$

$$n(E) = 200$$

It can be written as

$$n(H \cup E) = n(H) + n(E) - n(H \cap E)$$

By substituting the values

$$400 = 250 + 200 - n(H \cap E)$$

By further calculation

$$400 = 450 - n(H \cap E)$$

So we get

$$n(H \cap E) = 450 - 400$$

$$n(H \cap E) = 50$$

Therefore, 50 people can speak both Hindi and English.

4. If S and T are two sets such that S has 21 elements, T has 32 elements, and S \cap T has 11 elements, how many elements does S \cup T have?

Solution:

We know that

$$n(S) = 21$$

$$n(T) = 32$$

$$n(S \cap T) = 11$$

It can be written as

$$n(S \cup T) = n(S) + n(T) - n(S \cap T)$$

Substituting the values

$$n(S \cup T) = 21 + 32 - 11$$

So we get

$$n(S \cup T) = 42$$

Therefore, the set (S \cup T) has 42 elements.

5. If X and Y are two sets such that X has 40 elements, X \cup Y has 60 elements and X \cap Y has 10 elements, how many elements does Y have?

Solution:

We know that

$$n(X) = 40$$

$$n(X \cup Y) = 60$$

$$n(X \cap Y) = 10$$

It can be written as

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

By substituting the values

$$60 = 40 + n(Y) - 10$$

On further calculation

$$n(Y) = 60 - (40 - 10) = 30$$

Therefore, the set Y has 30 elements.

6. In a group of 70 people, 37 like coffee, 52 like tea, and each person likes at least one of the two drinks. How many people like both coffee and tea?

Solution:

Consider C as the set of people who like coffee

T as the set of people who like tea

$$n(C \cup T) = 70$$

$$n(C) = 37$$

$$n(T) = 52$$

It is given that

$$n(C \cup T) = n(C) + n(T) - n(C \cap T)$$

Substituting the values

$$70 = 37 + 52 - n(C \cap T)$$

By further calculation

$$70 = 89 - n(C \cap T)$$

So we get

$$n(C \cap T) = 89 - 70 = 19$$

Therefore, 19 people like both coffee and tea.

7. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Solution:

Consider C as the set of people who like cricket

T as the set of people who like tennis

$$n(C \cup T) = 65$$

$$n(C) = 40$$

$$n(C \cap T) = 10$$

It can be written as

$$n(C \cup T) = n(C) + n(T) - n(C \cap T)$$

Substituting the values

$$65 = 40 + n(T) - 10$$

By further calculation

$$65 = 30 + n(T)$$

So we get

$$n(T) = 65 - 30 = 35$$

Hence, 35 people like tennis.

We know that,

$$(T - C) \cup (T \cap C) = T$$

So we get,

$$(T - C) \cap (T \cap C) = \Phi$$

Here

$$n(T) = n(T - C) + n(T \cap C)$$

Substituting the values

$$35 = n(T - C) + 10$$

By further calculation

$$n(T - C) = 35 - 10 = 25$$

Therefore, 25 people like only tennis.

8. In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

Solution:

Consider F as the set of people in the committee who speak French

S as the set of people in the committee who speak Spanish

$$n(F) = 50$$

$$n(S) = 20$$

$$n(S \cap F) = 10$$

It can be written as

$$n(S \cup F) = n(S) + n(F) - n(S \cap F)$$

By substituting the values

$$n(S \cup F) = 20 + 50 - 10$$

By further calculation

$$n(S \cup F) = 70 - 10$$

$$n(S \cup F) = 60$$

Therefore, 60 people in the committee speak at least one of the two languages.

Miscellaneous Exercise Page 26

1. Decide, among the following sets, which sets are subsets of one and another:

$$A = \{x: x \in \mathbb{R} \text{ and } x \text{ satisfy } x^2 - 8x + 12 = 0\},$$

$$B = \{2, 4, 6\},$$

$$C = \{2, 4, 6, 8, \dots\},$$

$$D = \{6\}.$$

Solution:

According to the question,

We have,

$$A = \{x: x \in \mathbb{R} \text{ and } x \text{ satisfies } x^2 - 8x + 12 = 0\}$$

2 and 6 are the only solutions of $x^2 - 8x + 12 = 0$.

$$\text{Hence, } A = \{2, 6\}$$

$$B = \{2, 4, 6\}, C = \{2, 4, 6, 8, \dots\}, D = \{6\}$$

$$\text{Hence, } D \subset A \subset B \subset C$$

$$\text{Hence, } A \subset B, A \subset C, B \subset C, D \subset A, D \subset B, D \subset C$$

2. In each of the following, determine whether the statement is true or false.

If it is true, prove it. If it is false, give an example.

- (i) If $x \in A$ and $A \in B$, then $x \in B$
- (ii) If $A \subset B$ and $B \in C$, then $A \in C$
- (iii) If $A \subset B$ and $B \subset C$, then $A \subset C$
- (iv) If $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$
- (v) If $x \in A$ and $A \not\subset B$, then $x \in B$
- (vi) If $A \subset B$ and $x \notin B$, then $x \notin A$

Solution:

(i) False

According to the question,

$$A = \{1, 2\} \text{ and } B = \{1, \{1, 2\}, \{3\}\}$$

Now, we have,

$$2 \in \{1, 2\} \text{ and } \{1, 2\} \in \{1, \{1, 2\}, \{3\}\}$$

Hence, we get,

$$A \in B$$

We also know,

$$\{2\} \notin \{1, \{1, 2\}, \{3\}\}$$

(ii) False

According to the question

Let us assume that,

$$A \{2\}$$

Call :- + 91 9953771000

@ 1/354, SADAR BAZAR, DELHI, CANTT - 110010

$B = \{0, 2\}$

And, $C = \{1, \{0, 2\}, 3\}$

From the question,

$A \subset B$

Hence,

$B \in C$

But, we know,

$A \notin C$

(iii) True

According to the question

$A \subset B$ and $B \subset C$

Let us assume that,

$x \in A$

Then, we have,

$x \in B$

And,

$x \in C$

Therefore,

$A \subset C$

(iv) False

According to the question

$A \not\subset B$

Also,

$B \not\subset C$

Let us assume that,

$A = \{1, 2\}$

$B = \{0, 6, 8\}$

And,

$C = \{0, 1, 2, 6, 9\}$

$\therefore A \subset C$

(v) False

According to the question,

$x \in A$

Also,

$A \not\subset B$

Let us assume that,

$A = \{3, 5, 7\}$

Also,

$$B = \{3, 4, 6\}$$

We know that,

$$A \not\subset B$$

$$\therefore 5 \notin B$$

(vi) True

According to the question,

$$A \subset B$$

Also,

$$x \notin B$$

Let us assume that,

$$x \in A,$$

We have,

$$x \in B,$$

From the question,

We have, $x \notin B$

$$\therefore x \notin A$$

3. Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. show that $B = C$.

Solution:

According to the question,

$$A \cup B = A \cup C$$

And,

$$A \cap B = A \cap C$$

To show,

$$B = C$$

Let us assume,

$$x \in B$$

So,

$$x \in A \cup B$$

$$x \in A \cup C$$

Hence,

$$x \in A \text{ or } x \in C$$

When $x \in A$, then,

$$x \in B$$

$$\therefore x \in A \cap B$$

$$\text{As, } A \cap B = A \cap C$$

So, $x \in A \cap C$

$\therefore x \in A$ or $x \in C$

$x \in C$

$\therefore B \subset C$

Similarly, it can be shown that $C \subset B$

Hence, $B = C$

4. Show that the following four conditions are equivalent:

(i) $A \subset B$ (ii) $A - B = \emptyset$

(iii) $A \cup B = B$ (iv) $A \cap B = A$

Solution:

According to the question,

To prove, (i) \leftrightarrow (ii)

Here, (i) = $A \subset B$ and (ii) = $A - B = \emptyset$

Let us assume that $A \subset B$

To prove, $A - B \neq \emptyset$

Let $A - B \neq \emptyset$

Hence, there exists $X \in A$, $X \notin B$, but since $A \subset B$, it is not possible

$\therefore A - B = \emptyset$

And $A \subset B \Rightarrow A - B \neq \emptyset$

Let us assume that $A - B \neq \emptyset$

To prove: $A \subset B$

Let $X \in A$

So, $X \in B$ (if $X \notin B$, then $A - B \neq \emptyset$)

Hence, $A - B = \emptyset \Rightarrow A \subset B$

$\therefore (i) \leftrightarrow (ii)$

Let us assume that $A \subset B$

To prove, $A \cup B = B$

$\Rightarrow B \subset A \cup B$

Let us assume, $x \in A \cup B$

$\Rightarrow X \in A$ or $X \in B$

Taking Case I: $X \in B$

$A \cup B = B$

Taking Case II: $X \in A$

$\Rightarrow X \in B$ ($A \subset B$)

$\Rightarrow A \cup B \subset B$

Let $A \cup B = B$

Let us assume that $X \in A$

$\Rightarrow X \in A \cup B$ ($A \subset A \cup B$)

$\Rightarrow X \in B$ ($A \cup B = B$)

$\therefore A \subset B$

Hence, (i) \leftrightarrow (iii)

To prove (i) \leftrightarrow (iv)

Let us assume that $A \subset B$

$A \cap B \subset A$

Let $X \in A$

To prove, $X \in A \cap B$

Since, $A \subset B$ and $X \in B$

Hence, $X \in A \cap B$

$\Rightarrow A \subset A \cap B$

$\Rightarrow A = A \cap B$

Let us assume that $A \cap B = A$

Let $X \in A$

$\Rightarrow X \in A \cap B$

$\Rightarrow X \in B$ and $X \in A$

$\Rightarrow A \subset B$

\therefore (i) \leftrightarrow (iv)

\therefore (i) \leftrightarrow (ii) \leftrightarrow (iii) \leftrightarrow (iv)

Hence, proved

5. Show that if $A \subset B$, then $C - B \subset C - A$.

Solution:

To show,

$C - B \subset C - A$

According to the question,

Let us assume that x is any element such that $X \in C - B$

$\therefore x \in C$ and $x \notin B$

Since, $A \subset B$, we have,

$\therefore x \in C$ and $x \notin A$

So, $x \in C - A$

$\therefore C - B \subset C - A$

Hence, Proved.

6. Assume that $P(A) = P(B)$. Show that $A = B$

Solution:

To show,

$A = B$

According to the question,

$$P(A) = P(B)$$

Let x be any element of set A ,

$$x \in A$$

Since, $P(A)$ is the power set of set A , it has all the subsets of set A .

$$A \in P(A) = P(B)$$

Let C be an element of set B

For any $C \in P(B)$,

We have, $x \in C$

$$C \subset B$$

$$\therefore x \in B$$

$$\therefore A \subset B$$

Similarly, we have:

$$B \subset A$$

So, we get,

If $A \subset B$ and $B \subset A$

$$\therefore A = B$$

7. Is it true that for any sets A and B , $P(A) \cup P(B) = P(A \cup B)$? Justify your answer.

Solution:

It is not true that for any sets A and B , $P(A) \cup P(B) = P(A \cup B)$

Justification:

Let us assume,

$$A = \{0, 1\}$$

$$\text{And, } B = \{1, 2\}$$

$$\therefore A \cup B = \{0, 1, 2\}$$

According to the question,

We have,

$$P(A) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$$

$$P(B) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$$

$$\therefore P(A \cup B) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{1, 2\}, \{0, 2\}, \{0, 1, 2\}\}$$

Also,

$$P(A) \cup P(B) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{1, 2\}\}$$

$$\therefore P(A) \cup P(B) \neq P(A \cup B)$$

Hence, the given statement is false

8. Show that for any sets A and B ,
 $A = (A \cap B) \cup (A - B)$ and $A \cup (B - A) = (A \cup B)$

Call :- + 91 9953771000

@ 1/354, SADAR BAZAR, DELHI, CANTT - 110010

Solution:

To Prove,

$$A = (A \cap B) \cup (A - B)$$

Proof: Let $x \in A$

To show,

$$X \in (A \cap B) \cup (A - B)$$

In Case I,

$$X \in (A \cap B)$$

$$\Rightarrow X \in (A \cap B) \subset (A \cup B) \cup (A - B)$$

In Case II,

$$X \notin A \cap B$$

$$\Rightarrow X \notin B \text{ or } X \notin A$$

$$\Rightarrow X \notin B (X \notin A)$$

$$\Rightarrow X \notin A - B \subset (A \cup B) \cup (A - B)$$

$$\therefore A \subset (A \cap B) \cup (A - B) \text{ (i)}$$

It can be concluded that, $A \cap B \subset A$ and $(A - B) \subset A$

Thus, $(A \cap B) \cup (A - B) \subset A$ (ii)

Equating (i) and (ii),

$$A = (A \cap B) \cup (A - B)$$

We also have to show,

$$A \cup (B - A) \subset A \cup B$$

Let us assume,

$$X \in A \cup (B - A)$$

$$X \in A \text{ or } X \in (B - A)$$

$$\Rightarrow X \in A \text{ or } (X \in B \text{ and } X \notin A)$$

$$\Rightarrow (X \in A \text{ or } X \in B) \text{ and } (X \in A \text{ and } X \notin A)$$

$$\Rightarrow X \in (B \cup A)$$

$$\therefore A \cup (B - A) \subset (A \cup B) \text{ (iii)}$$

According to the question,

To prove:

$$(A \cup B) \subset A \cup (B - A)$$

Let $y \in A \cup B$

$$Y \in A \text{ or } y \in B$$

$$(y \in A \text{ or } y \in B) \text{ and } (X \in A \text{ and } X \notin A)$$

$$\Rightarrow y \in A \text{ or } (y \in B \text{ and } y \notin A)$$

$$\Rightarrow y \in A \cup (B - A)$$

$$\text{Thus, } A \cup B \subset A \cup (B - A) \text{ (iv)}$$

∴ From equations (iii) and (iv), we get:

$$A \cup (B - A) = A \cup B$$

9. Using properties of sets, show that:

$$(i) A \cup (A \cap B) = A$$

$$(ii) A \cap (A \cup B) = A.$$

Solution:

$$(i) \text{ To show: } A \cup (A \cap B) = A$$

We know that,

$$A \subset A$$

$$A \cap B \subset A$$

$$\therefore A \cup (A \cap B) \subset A \text{ (i)}$$

Also, according to the question,

We have:

$$A \subset A \cup (A \cap B) \text{ (ii)}$$

Hence, from equation (i) and (ii)

We have:

$$A \cup (A \cap B) = A$$

(ii) To show,

$$A \cap (A \cup B) = A$$

$$A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$$

$$= A \cup (A \cap B)$$

$$= A$$

10. Show that $A \cap B = A \cap C$ need not imply $B = C$.

Solution:

Let us assume,

$$A = \{0, 1\}$$

$$B = \{0, 2, 3\}$$

$$\text{And, } C = \{0, 4, 5\}$$

According to the question,

$$A \cap B = \{0\}$$

And,

$$A \cap C = \{0\}$$

$$\therefore A \cap B = A \cap C = \{0\}$$

But,

$2 \in B$ and $2 \notin C$

Therefore, $B \neq C$

11. Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that $A = B$.

(Hints $A = A \cap (A \cup X)$, $B = B \cap (B \cup X)$ and use Distributive law)

Solution:

According to the question,

Let A and B be two sets such that $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X.

To show, $A = B$

Proof:

$$A = A \cap (A \cup X) = A \cap (B \cup X) \quad [A \cup X = B \cup X]$$

$$= (A \cap B) \cup (A \cap X) \quad [\text{Distributive law}]$$

$$= (A \cap B) \cup \phi \quad [A \cap X = \phi]$$

$$= A \cap B \quad (\text{i})$$

$$\text{Now, } B = B \cap (B \cup X)$$

$$= B \cap (A \cup X) \quad [A \cup X = B \cup X]$$

$$= (B \cap A) \cup (B \cap X) \dots \quad [\text{Distributive law}]$$

$$= (B \cap A) \cup \phi \quad [B \cap X = \phi]$$

$$= A \cap B \quad (\text{i})$$

Hence, from equations (i) and (ii), we obtain $A = B$.

12. Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \phi$.

Solution:

Let us assume, $A = \{0, 1\}$

$B = \{1, 2\}$

And, $C = \{2, 0\}$

According to the question,

$$A \cap B = \{1\}$$

$$B \cap C = \{2\}$$

And,

$$A \cap C = \{0\}$$

$\therefore A \cap B$, $B \cap C$ and $A \cap C$ are not empty sets

Hence, we get,

$$A \cap B \cap C = \phi$$

13. In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee.

Solution:

Call :- + 91 9953771000

@ 1/354, SADAR BAZAR, DELHI, CANTT - 110010

Let us assume that,

U = the set of all students who took part in the survey

T = the set of students taking tea

C = the set of the students taking coffee

Total number of students in a school, $n(U) = 600$

Number of students taking tea, $n(T) = 150$

Number of students taking coffee, $n(C) = 225$

Also, $n(T \cap C) = 100$

Now, we have to find that number of students taking neither coffee nor tea i.e. $n(T \cap C')$

\therefore According to the question,

$$n(T \cap C') = n(T \cap C)'$$

$$= n(U) - n(T \cap C)$$

$$= n(U) - [n(T) + n(C) - n(T \cap C)]$$

$$= 600 - [150 + 225 - 100]$$

$$= 600 - 275$$

$$= 325$$

\therefore Number of students taking neither coffee nor tea = 325 students

14. In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

Solution:

Let us assume that,

U = the set of all students in the group

E = the set of students who know English

H = the set of the students who know Hindi

$$\therefore H \cup E = U$$

Given that,

Number of students who know Hindi $n(H) = 100$

Number of students who knew English, $n(E) = 50$

Number of students who know both, $n(H \cap E) = 25$

We have to find the total number of students in the group i.e. $n(U)$

\therefore According to the question,

$$n(U) = n(H) + n(E) - n(H \cap E)$$

$$= 100 + 50 - 25$$

$$= 125$$

\therefore Total number of students in the group = 125 students

15. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:

- The number of people who read at least one of the newspapers.
- The number of people who read exactly one newspaper.

Solution:

- Let us assume that,

A = the set of people who read newspaper H

B = the set of people who read newspaper T

C = the set of people who read newspaper I

According to the question,

Number of people who read newspaper H, $n(A) = 25$

Number of people who read newspaper T, $n(B) = 26$

Number of people who read the newspaper I, $n(C) = 26$

Number of people who read both newspaper H and I, $n(A \cap C) = 9$

Number of people who read both newspaper H and T, $n(A \cap B) = 11$

Number of people who read both newspaper T and I, $n(B \cap C) = 8$

And, Number of people who read all three newspaper H, T and I, $n(A \cap B \cap C) = 3$

Now, we have to find the number of people who read at least one of the newspaper

\therefore we get.

$$\begin{aligned}
 n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + \\
 &\quad n(A \cap B \cap C) \\
 &= 25 + 26 + 26 - 11 - 8 - 9 + 3 \\
 &= 80 - 28 \\
 &= 52
 \end{aligned}$$

\therefore There are a total of 52 students who read at least one newspaper.

- Let us assume that,

a = the number of people who read newspapers H and T only

b = the number of people who read newspapers I and H only

c = the number of people who read newspapers T and I only

d = the number of people who read all three newspapers

According to the question,

$$D = n(A \cap B \cap C) = 3$$

Now, we have:

$$n(A \cap B) = a + d$$

$$n(B \cap C) = c + d$$

And,

$$n(C \cap A) = b + d$$

$$\therefore a + d + c + d + b + d = 11 + 8 + 9$$

$$a + b + c + d = 28 - 2d$$

$$= 28 - 6$$

$$= 22$$

$$\therefore \text{Number of people read exactly one newspaper} = 52 - 22$$

$$= 30 \text{ people}$$



LOYAL Education

<https://loyaleducation.org>